

# Lepton-flavor mixing and $K \rightarrow \pi \nu \bar{\nu}$ decays

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## Abstract

The impact of possible sources of lepton-flavor mixing on  $K \rightarrow \pi \nu \bar{\nu}$  decays is analysed. At the one-loop level lepton-flavor mixing originated from non-diagonal lepton mass matrices cannot generate a CP-conserving  $K_L \rightarrow \pi^0 \nu \bar{\nu}$  amplitude. The rates of these modes are sensitive to leptonic flavor violation when there are at least two different leptonic mixing matrices. New interactions that violate both quark and lepton universalities could enhance the CP-conserving component of  $\Gamma(K_L \rightarrow \pi^0 \nu \bar{\nu})$  and have a substantial impact. Explicit examples of these effects in the context of supersymmetric models, with and without  $R$ -parity conservation, are discussed.

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# 1 Introduction

Within the Standard Model (SM), the Flavor-Changing-Neutral-Current (FCNC) decays  $K \rightarrow \pi \nu \bar{\nu}$  are among the cleanest observables to determine the mixing of the top quark with the light generations. In particular, the  $K_L \rightarrow \pi^0 \nu \bar{\nu}$  rate is completely dominated by a CP-violating (CPV) amplitude and could be used to determine with high precision the Jarlskog's invariant [1, 2]. The situation could be very different beyond the SM: similarly to all FCNC transitions,  $K \rightarrow \pi \nu \bar{\nu}$  decays are highly sensitive to new sources of quark-flavor mixing. However, a peculiar aspect of these decays is their potential sensitivity also to flavor mixing in the leptonic sector. The most remarkable consequence of this fact is that the transition  $K_L \rightarrow \pi^0 \nu_i \bar{\nu}_j$ , with  $i \neq j$ , does not need to be dominated by a CPV amplitude [3].

Recent results from neutrino physics indicate that the quark and lepton sectors have a rather different flavor structure. In particular, we now know that large mixing angles do appear in the lepton sector. Due to the smallness of neutrino masses, these large mixing angles have no impact on  $K \rightarrow \pi \nu \bar{\nu}$  rates in minimal models, where only neutrino mass terms are introduced [4]. However, this conclusion is not necessarily true in more general scenarios, such as supersymmetric models, with possible large mixing angles also in the slepton sector.

In this letter we present a general analysis of the impact of lepton-flavor mixing on  $K \rightarrow \pi \nu \bar{\nu}$  decays. As we shall show, if left-handed neutrinos are the only light fields and lepton-flavor mixing is confined only to mass matrices, lepton-flavor violation cannot be the dominant effect on the  $K \rightarrow \pi \nu \bar{\nu}$  rates. In particular, it cannot induce a CPC  $K_L \rightarrow \pi^0 \nu \bar{\nu}$  amplitude. This conclusion is independent of the type of mass matrices involved (e.g., neutrinos, sneutrinos, leptons or sleptons). However, if more than one mass matrix is involved, the effect of lepton-flavor mixing is not necessarily negligible. We demonstrate it in the Minimal Supersymmetric SM (MSSM), where the charged-slepton-neutrino and the sneutrino-neutrino mixing matrices are in general different. In order to induce a non-negligible CPC  $K_L \rightarrow \pi^0 \nu \bar{\nu}$  transition, lepton-flavor mixing in mass matrices is not sufficient and we need a new interaction that violates both quark and lepton universality. We illustrate this effect with two examples of non-universal interactions: the lepton-quark-squark coupling in the framework of the  $R$ -parity violating MSSM and the Yukawa interaction in the  $R$ -parity conserving MSSM.

## 2 General properties of $K \rightarrow \pi \nu \bar{\nu}$ amplitudes

The SM contributions to  $K \rightarrow \pi \nu \bar{\nu}$  amplitudes are described by the following effective Hamiltonian [5]

$$\mathcal{H}_{\text{eff}}^{\text{SM}} = \frac{4G_F}{\sqrt{2}} \frac{\alpha}{2\pi \sin^2 \Theta_W} \sum_{\ell=e,\mu,\tau} \left[ \lambda_c X_{NL}^\ell + \lambda_t X(x_t) \right] \bar{s}_L \gamma^\mu d_L \times \bar{\nu}_L^\ell \gamma_\mu \nu_L^\ell + \text{h.c.} , \quad (1)$$

where  $x_t = m_t^2/M_W^2$ ,  $\lambda_q = V_{qs}^* V_{qd}$  and  $V_{ij}$  denote CKM matrix elements. The coefficients  $X_{NL}^\ell$  and  $X(x_t)$ , encoding top- and charm-quark loop contributions, are known at the

NLO accuracy in QCD [5, 6] leading to a very precise prediction of the decay rates. Note that the dependence on the lepton flavor that enter via  $X_{NL}^\ell$  is very small, and we neglect it in the following. The neutrino pair produced by  $\mathcal{H}_{\text{eff}}^{\text{SM}}$  is a CP eigenstate with positive eigenvalue. This is the reason why the leading SM contribution to  $K_L \rightarrow \pi^0 \nu_i \bar{\nu}_i$  is due to CP violation [1]. Within the SM, CP-Conserving (CPC) contributions to  $K_L \rightarrow \pi^0 \nu_i \bar{\nu}_i$  are generated only by local operators of dimension  $d \geq 8$  or by long-distance effects: these contributions do not exceed the  $10^{-4}$  level in the total rate, compared to the dominant CP-violating term [7].

The situation could be very different beyond the SM, where new dimension-six operators could contribute to  $K \rightarrow \pi \nu \bar{\nu}$  amplitudes. In principle, beyond the SM one should also take into account other  $K \rightarrow \pi + X_{\text{invisible}}$  transitions, which could lead to similar experimental signatures. In order to classify the relevant operators, it is necessary to specify which are the light invisible degrees of freedom of the theory, and what are their interactions. For our purpose, we can distinguish three main scenarios:

1. *The only light invisibles are the three species of left-handed neutrinos.*

In this case the only relevant dimension-six operators are:

$$O_{sd}^{ij} = \bar{s} \gamma_\mu d \times \bar{\nu}_L^i \gamma^\mu \nu_L^j . \quad (2)$$

For  $i \neq j$  these operators create a neutrino pair which is not a CP eigenstate. In principle, one can also write operators of the type  $(\bar{s} \Gamma d) \times \nu_L^C \Gamma \nu_L$ , which break both lepton-number and  $SU(2)_L$ -invariance. As expected by this highly-breaking structure, and as explicitly shown in Ref. [4], the effect of these additional operators is completely negligible.

2. *Right-handed neutrinos are also light, but they are sterile.*

In this case we need to consider also scalar and tensor dimension-six operators of the type  $(\bar{s} \Gamma d) \times \bar{\nu}_{R(L)} \Gamma \nu_{L(R)}$ ; however, if right-handed neutrinos are sterile the coupling of these operators is negligible. An explicit realization of this scenarios occurs in all the models where the right-handed neutrinos interact with the SM fields only through their (tiny) Dirac mass terms [4].

3. *Right-handed neutrinos are light and not sterile.*

If the right-handed neutrino fields are not sterile, the coupling of the scalar and tensor operators mentioned above (case 2) is not necessarily suppressed and these operators could compete with the leading left-handed terms in (2). This occurs for instance in LR symmetric models, where the right-handed neutrino fields couple to quarks via new gauge interactions [8]. In this framework lepton-flavor mixing could have a non-negligible impact on  $K \rightarrow \pi \nu \bar{\nu}$  rates. The scalar and tensor operators have a different CP structure with respect to the SM operator and they induce a CPC contribution to  $K_L \rightarrow \pi^0 \nu \bar{\nu}$  in absence of lepton-flavor mixing [8].

An important difference of the last two cases with respect to the first one is the fact that scalar and tensor operators would also lead to a different pion-energy spectrum. Thus, the

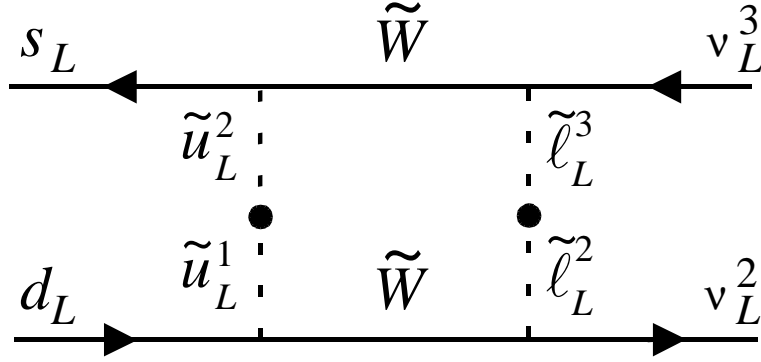


Figure 1: Wino-Wino box diagram.

first case is in principle distinguishable from the last two by means of experimental data. This conclusion can be generalized to most of the other  $K \rightarrow \pi + X_{\text{invisible}}$  transitions, where  $X_{\text{invisible}}$  include other degrees of freedom in addition to the neutrinos.<sup>‡</sup>

In the following, we shall analyze in more detail the effect of lepton-flavor mixing in the first case above, when only the operators (2) are relevant, and then the pion-energy spectrum is identical to the SM case.

### 3 Lepton-flavor mixing in mass matrices

Since the  $\nu^j \bar{\nu}^i$  final state is not a CP eigenstate, the condition for a non-vanishing  $K_L \rightarrow \pi^0 \nu \bar{\nu}$  rate seems to be the breaking of CP or lepton-flavor symmetries. As we explain below, the condition turns out to be stronger: we need either CP violation in the quark sector or a new effective interaction that violates both quark and lepton universality.

If the breaking of flavor universality can be confined only to appropriate mass matrices, both in the quark and in the lepton sectors, and the two sectors are connected by flavor-universal interactions, quark- and lepton-flavor mixing terms in  $K \rightarrow \pi \nu \bar{\nu}$  amplitudes assume a factorizable structure. In this case we can always rotate the neutrino eigenstates to diagonalize the lepton final state, without any impact on the quark structure. As a result, the inclusive sum over neutrino flavors can be transformed into a sum over CP eigenstates. It is then clear that the  $K_L \rightarrow \pi^0 \nu \bar{\nu}$  transition vanishes in absence of CP violation in the quark sector.

We note the following two points:

1. Even with the factorizable structure, the (lepton) mass matrices may have impact on  $K \rightarrow \pi \nu \bar{\nu}$  rates. The eigenvalues of the mass matrices are certainly relevant and, if more than one non-trivial mass matrix is involved, also their relative rotation angles can play a significant role.
2. The factorization structure is expected to be broken by higher-order loop effects.

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<sup>‡</sup>For example, there is a possible decay  $K \rightarrow \pi f$  where  $f$  is a “familon,” a Nambu–Goldstone boson of the spontaneously broken horizontal symmetry. However, this process can be discriminated experimentally because of the two-body kinematics.

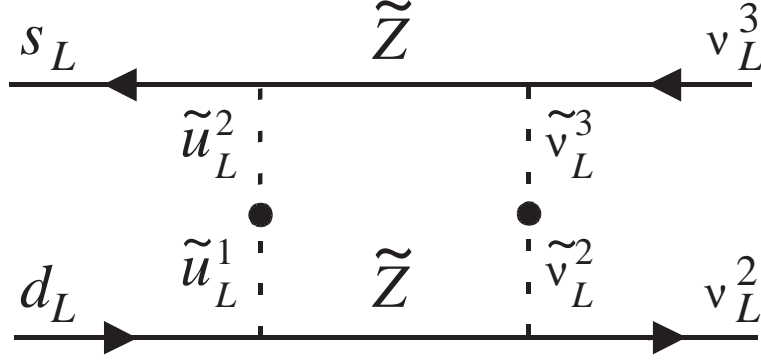


Figure 2: Zino-Zino box diagram.

Then, the flavor breaking in the mass terms could induce a breaking of universality also in effective interaction vertices. Since this is a higher-order effect, it is likely to be highly suppressed.

To illustrate the above argument, we discuss a specific example of a factorizable structure: the one originated from the  $\tilde{W}$ -box diagrams in Fig. 1. Using the fact that the weak interaction is universal, we can write the decay amplitude in the basis where squark and slepton mass matrices are diagonal as

$$A(K^0 \rightarrow \pi \nu_i \bar{\nu}_j) = \frac{1}{\sqrt{2}} \sum_{q,\ell} \hat{V}_{sq} \hat{V}_{dq}^* \hat{U}_{i\ell} \hat{U}_{j\ell}^* f(m_{\tilde{q}}, m_{\tilde{\ell}}) . \quad (3)$$

Here  $f(m_{\tilde{q}}, m_{\tilde{\ell}})$  is the loop function, which depends on squark and slepton masses.  $\hat{V}$ ,  $[\hat{U}]$  is a unitary matrix describing the rotation from the electroweak (interaction) eigenstates to the mass eigenstates in the  $\tilde{W} \tilde{u}_i d_j$  [ $\tilde{W} \tilde{\ell}_i \nu_j$ ] interaction. Working in the basis where  $CP|K^0\rangle = |\bar{K}^0\rangle$  we get

$$A(\bar{K}^0 \rightarrow \pi \nu_i \bar{\nu}_j) = \frac{1}{\sqrt{2}} \sum_{q,\ell} \hat{V}_{sq}^* \hat{V}_{dq} \hat{U}_{i\ell} \hat{U}_{j\ell}^* f(m_{\tilde{q}}, m_{\tilde{\ell}}) . \quad (4)$$

Introducing the diagonal matrix  $\mathcal{F}_q$ , defined by  $(\mathcal{F}_q)_{ii} = f(m_q, m_i)$ , the  $K_L \rightarrow \pi^0 \nu_i \bar{\nu}_j$  amplitude can be written as

$$A(K_L \rightarrow \pi \nu_i \bar{\nu}_j) = i \sum_q \text{Im} \left( \hat{V}_{sq} \hat{V}_{dq}^* \right) \left[ \hat{U} \mathcal{F}_q \hat{U}^\dagger \right]_{ij} . \quad (5)$$

We see that the amplitude vanishes if there is no CP-violation in the quark sector.

Assuming that the full amplitude is given by eq. (5) and ignoring phase-space effects due to non-vanishing neutrino masses, the total rate obtained by summing over neutrino flavors is given by

$$\begin{aligned} \Gamma(K_L \rightarrow \pi^0 \nu \bar{\nu}) &\propto \sum_{ij} |A(K_L \rightarrow \pi \nu_i \bar{\nu}_j)|^2 \\ &= \sum_{q,k} \text{Im} \left( \hat{V}_{sq} \hat{V}_{dq}^* \right) \text{Im} \left( \hat{V}_{sk} \hat{V}_{dk}^* \right) \text{tr} \left[ \hat{U} \mathcal{F}_q \hat{U}^\dagger \hat{U} \mathcal{F}_k \hat{U}^\dagger \right] \\ &= \sum_{q,k} \text{Im} \left( \hat{V}_{sq} \hat{V}_{dq}^* \right) \text{Im} \left( \hat{V}_{sk} \hat{V}_{dk}^* \right) \text{tr} [\mathcal{F}_q \mathcal{F}_k] . \end{aligned} \quad (6)$$

We see that the lepton-flavor mixing matrix  $\hat{U}$  disappears from the trace over lepton indices. This is a result of the fact that we sum over all the final-state neutrino flavors. On the other hand, the eigenvalues of the slepton mass matrix enter in the determination of  $\text{tr}[\mathcal{F}_q \mathcal{F}_k]$ .

Similar arguments hold also for the SM with massive neutrinos [4]. In that case, as well as in our more general case, the  $K_L$  decay amplitude arises only due to CP violation in the quark (or squark) sector.

Now we consider a case where we have two different amplitudes with different flavor mixing. For example, we add the  $\tilde{Z}$ -box diagrams of Fig. 2. Similarly to the wino diagram the amplitude is given by

$$A(K_L \rightarrow \pi \nu_i \bar{\nu}_j) = i \sum_q \text{Im} \left( \hat{V}'_{sq} \hat{V}_{dq}^* \right) \left[ \hat{U}' \mathcal{G}_q \hat{U}^{\prime\dagger} \right]_{ij} . \quad (7)$$

where  $\mathcal{G}_q$  is defined similar to  $\mathcal{F}_q$  and the primed matrices are the ones that rotate the neutral interaction. In general,  $V \neq V'$  and  $U \neq U'$ . Adding the two amplitudes and neglecting the SM contribution we get

$$\Gamma(K_L \rightarrow \pi^0 \nu \bar{\nu}) \propto \sum_{q,k} \left\{ a_F^q a_F^k \text{tr} [\mathcal{F}_q \mathcal{F}_k] + a_G^q a_G^k \text{tr} [\mathcal{G}_q \mathcal{G}_k] + 2 a_F^q a_G^k \text{tr} [W^\dagger \mathcal{F}_q W \mathcal{G}_k] \right\} , \quad (8)$$

where

$$a_F^q = \text{Im} \left( \hat{V}_{sq} \hat{V}_{dq}^* \right) , \quad a_G^q = \text{Im} \left( \hat{V}'_{sq} \hat{V}_{dq}^* \right) , \quad (9)$$

and

$$W \equiv \hat{U}^\dagger \hat{U}' . \quad (10)$$

We see that the product of the mixing matrix enter in the interference term.

We note the following points:

1. When  $\mathcal{F}_q$  or  $\mathcal{G}_q$  are proportional to the unit matrix there is no sensitivity to the mixing matrix  $W$ . This is the case when the charged sleptons or sneutrinos are degenerate. More generally, we conclude that the effect is suppressed by the amount of degeneracy in the slepton sector.
2. The effect of the leptonic mixing cannot be very large. Since  $W$  is unitary, we learn that it is at most an  $O(1)$  effect. Yet, the effect can be large enough to be detectable.

## 4 Flavor non-universal interactions

### 4.1 $R$ -parity violating SUSY

A typical example of interaction that violates both quark and lepton universality is a family non-universal leptoquark (LQ). In  $R$ -parity violating supersymmetric models, the squarks, which couples to quark and leptons via the  $R$ -parity breaking  $LQ\bar{d}$  term, provides an explicit example of this scenario. In this context, the CPC  $K_L \rightarrow \pi^0 \nu_i \bar{\nu}_j$  transition mediated by operators of the type (2) is generated already at tree level. To illustrate the general conditions under which the CPC rate can be large, we shall discuss the LQ example in more detail.

Consider the following interaction term

$$\lambda_{\ell q} \bar{q}_L^c \ell_L S, \quad (11)$$

where  $S$  is a scalar LQ and the other notations are clear. This leads to the following effective dimension-six Hamiltonian [9]

$$\begin{aligned} \mathcal{H}_{\text{eff}}^{\text{LQ}} = & \frac{1}{M_S^2} \left\{ \bar{s} \gamma_\mu d \left[ \lambda_{is} \lambda_{jd}^* \bar{\nu}_L^i \gamma^\mu \nu_L^j + \lambda_{js} \lambda_{id}^* \bar{\nu}_L^j \gamma^\mu \nu_L^i \right] \right. \\ & \left. + \bar{d} \gamma_\mu s \left[ \lambda_{id} \lambda_{js}^* \bar{\nu}_L^i \gamma^\mu \nu_L^j + \lambda_{jd} \lambda_{is}^* \bar{\nu}_L^j \gamma^\mu \nu_L^i \right] \right\}. \end{aligned} \quad (12)$$

We then obtain

$$A(K_L \rightarrow \pi \nu_i \bar{\nu}_j) \propto (\lambda_{is} \lambda_{jd}^* - \lambda_{id} \lambda_{js}^*). \quad (13)$$

We note the following points:

1. In the general case there is no lepton and quark factorization. Then the  $K_L$  amplitude does not vanish, and, for  $i \neq j$ , contains both CPV and CPC terms.
2. In a specific scenario where the LQ coupling is universal with respect to the lepton flavor, namely  $\lambda_{iq} = \lambda_{jq}$  for each  $q$ , the amplitude is proportional to  $\text{Im}(\lambda_{is} \lambda_{id}^*)$ . In this case the amplitude is purely CP violating where, similarly to the SM case, the CP violation originates from the quark sector.
3. If the LQ coupling is universal with respect to the quark flavor, namely  $\lambda_{is} = \lambda_{id}$  for each  $i$ , the amplitude vanishes. This is expected since quark mixing is necessary for any FCNC process.

In the case of quarks and leptons of the first two generations, the interaction term in (11) is severely constrained by  $\pi$  and  $K$  semileptonic decays. Nonetheless, the strongest bound on off-diagonal combinations like  $\lambda_{2s}^* \lambda_{3d}$  come from  $B(K^+ \rightarrow \pi^+ \nu \bar{\nu})$  [3, 10]. Therefore, tuning appropriately these parameters one can generate a huge CPC conserving transition of the type  $K_L \rightarrow \pi^0 \nu_3 \bar{\nu}_2 + \pi^0 \nu_2 \bar{\nu}_3$ . As mentioned before, this occurs only when  $\lambda_{2s} \lambda_{3d}^* \neq \lambda_{2d} \lambda_{3s}^*$ . In that case the final state is not a CP eigenstate and thus the decay is a combination of CPC and CPV transitions.

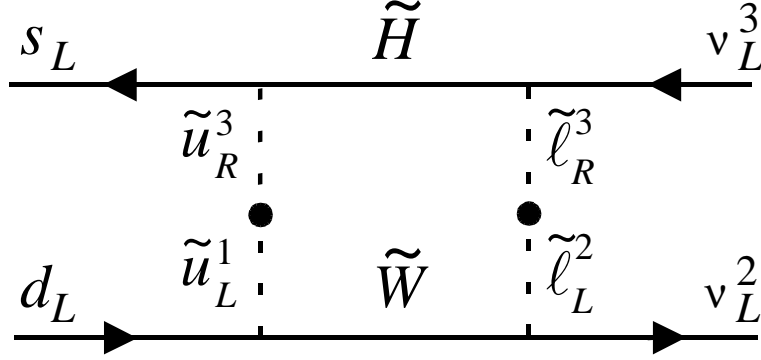


Figure 3: Wino-Higgsino box diagram

As shown in [3], also in this scenario the  $K_L$  width (summed over neutrino flavors) cannot exceed in magnitude the  $K^+$  one. In view of the recent BNL-E787 result on the charged mode [11], this model-independent relation implies  $\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu}) < 1.7 \times 10^{-9}$  (90% C.L.).

## 4.2 $R$ -parity conserving SUSY

In less exotic scenarios, like the SM with massive neutrinos or the MSSM with  $R$ -parity conservation, the only interaction that violates quark and lepton universality is the Yukawa interaction. Therefore, within these models the CPC contributions to  $K_L \rightarrow \pi^0 \nu_i \bar{\nu}_j$  are necessarily suppressed by Yukawa couplings. In the SM these terms are absolutely negligible [4]. The situation, however, is less obvious in the MSSM. There, the Yukawa couplings of the lepton (for  $\tan \beta \gg 1$ ) and both the slepton- and squark-flavor mixing angles can be large.

The potentially largest CPC contribution is generated from the non-universal interaction in Fig. 3. Contrary to the case of Fig. 1, here the exchange  $s \leftrightarrow d$  cannot be simply re-absorbed into the phase of the quark-mixing term. From the point of view of the low-energy effective Hamiltonian, this diagram is equivalent to a LQ exchange with

$$\lambda_{2s} \lambda_{3d}^* \propto y_t y_\tau V_{ts}^* (\delta_{LR}^{U*})_{13} (\delta_{LR}^L)_{23}. \quad (14)$$

Similar contribution arises for the  $(\nu_3 \bar{\nu}_3)$  final state, but then the amplitude is proportional to  $\text{Im}(V_{ts}^* (\delta_{LR}^{U*})_{13})$  and the effect is purely CPV.

Considering only the flavor violating contribution and using the results of Refs. [12, 13] we obtain

$$\begin{aligned} \frac{\Gamma(K_L \rightarrow \pi^0 \nu_3 \bar{\nu}_2 + \pi^0 \nu_2 \bar{\nu}_3)_{\text{MSSM}}}{\Gamma(K_L \rightarrow \pi^0 \nu \bar{\nu})_{\text{SM}}} &= \frac{1}{3} \left( \frac{m_\tau m_t \tan \beta}{8 M_W^2} \right)^2 \left| \frac{(\delta_{LR}^U)_{13} (\delta_{LR}^L)_{23}}{\text{Im}(V_{td})_{\text{SM}}} \right|^2 \left| \frac{F_{\text{loop}}(x_{ij})}{X_t(x_t)} \right|^2 \\ &\lesssim \left( \frac{\tan \beta}{50} \right)^2 \left| (\delta_{LR}^U)_{13} (\delta_{LR}^L)_{23} \right|^2. \end{aligned} \quad (15)$$

As usual  $(\delta_{LR}^A)_{ij} = (\tilde{M}_A^2)_{i_L j_R} / (\tilde{M}_A^2)_{i_L i_L}$  denote off-diagonal entries of squark and lepton mass matrices. The dimensionless loop function  $F_{\text{loop}}(x_{ij})$ , which depends on the ratio of



sparticle masses ( $x_{ij} = m_i^2/m_j^2$ ) is very small:

$$F_{\text{loop}}(x_{ij}) = x_{q_L\chi_1} x_{\ell_L\chi_1} k(x_{q_L\chi_1}, x_{q_R\chi_1}, x_{\ell_L\chi_1}, x_{\ell_R\chi_1}, x_{\chi_2\chi_1}) \longrightarrow \frac{1}{30} \quad (\text{for } x_{ij} = 1) , \quad (16)$$

with  $k$  defined as in [13]. This confirms the observation of Ref. [12] that SUSY box-diagram contributions to  $K \rightarrow \pi\nu\bar{\nu}$  are suppressed. The upper figure ( $F_{\text{loop}} \approx 0.05$ ) is obtained with a large splitting between left-handed and right-handed sfermions.

Given the bounds on the left-right mass insertions of squarks [12, 13] and leptons [14], we conclude that the ratio in Eq. (15) cannot exceed the  $10^{-2}$  level. If this bound were saturated, this CPC contribution would be much larger than the SM one, but of course would still be negligible compared to the SM CP-violating rate (and thus undetectable).

## 5 Conclusions

$K \rightarrow \pi\nu\bar{\nu}$  decays are certainly one of the cleanest windows to the short-distance mechanism of quark-flavor mixing. The result of the BNL-E787 Collaboration [11], although still affected by a large experimental error, already shows the great potential of these modes in constraining flavor physics within and beyond the SM [16].

Beside the obvious sensitivity to quark-flavor mixing,  $K \rightarrow \pi\nu\bar{\nu}$  decays are in principle affected also by mixing of lepton flavors [3]. In this letter we have investigated under which conditions the leptonic mixing can play a significant role in these modes. First we studied the case where the sources of quark- and lepton-flavor mixing can be factorized. In particular, we concentrate on cases where the source of flavor-symmetry breaking is confined to mass matrices, since then this factorization is almost complete. We found that the sum over neutrino flavors (implicitly understood in  $K \rightarrow \pi\nu\bar{\nu}$  rates) wash out any individual effect due to lepton-flavor mixing. Only in cases where there are two different lepton-flavor mixing matrices, there is an effect which depends on the product of the two mixing matrices. Then we studied interactions that violate at the same time quark and lepton universality. In that case individual leptonic flavor violation can be important as they induce CPC contribution to the rate.

In models like the SM or the  $R$ -parity conserving MSSM, but also in models with large extra dimensions with a protective flavor symmetry (see e.g. Ref. [15]), only the Yukawa interaction violates at the same time quark and lepton universality. In these models CPC lepton-flavor mixing effects in  $K \rightarrow \pi\nu\bar{\nu}$  decays are therefore suppressed by Yukawa couplings. As we have explicitly shown, even in a very favorable case, such as the MSSM with generic flavor couplings and large  $\tan\beta$ , these types of CPC lepton-flavor mixing effects are negligible. In more exotic scenarios, such as  $R$ -parity violating supersymmetric models, lepton-flavor mixing could generate significant effects in  $K \rightarrow \pi\nu\bar{\nu}$  decays, in particular, a sizable  $K_L \rightarrow \pi^0\nu\bar{\nu}$  CP-conserving rate.

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